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MAGNETOTEMPERATURE WAVES IN ELECTRICALLY CONDUCTING MEDIA

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A new type of temperature waves that appear in an electrically conducting medium placed in a magnetostatic field is discovered. The velocity of these magnetotemperature waves is determined by the coefficient of electrical conductivity λ of the conductor and greatly exceeds the velocity of temperature propagation by thermal conduction.

I. The majority of texts on electromagnetism consider in detail phenomena that occur in metallic specimens placed in an external electric field, and it is emphasized that free charges of the metal appear instantly on its surface, so that the external force lines, being deformed, are normal to it, and the electric field inside the conductor is always equal to zero [1-4].

However, the same publications fail to mention anything about what occurs in a conductor placed in a steady magnetic field. Analysis of this process shows that in the latter case the phenomena differ substantially from those occurring in an electrostatic field.

First, the external magnetic field penetrates, though not very rapidly, inside the conductor; second, penetration by the magnetic field is accompanied by motion from the conductor surface to its center of a kind of heat wave, which will be called a *magnetotemperature wave.* The front of this wave represents a closed surface, i.e., a two-dimensional soliton whose height is determined by the temperature of a moving thin layer.

Let us move to a quantitative consideration of the effect. As is known from electrodynamics [1], the boundary conditions for a magnetostatic field have the form:

$$B_{1n} - B_{2n} = 0\,, \tag{1}$$

$$B_{1\tau} - B_{2\tau} = \frac{4\pi}{C} j'\,. \tag{2}$$

If we take into account that at the initial time instant $t = 0$ a magnetic field in the conductor is absent, the boundary conditions are simplified to

$$B_{1n} = 0\,, \tag{1'}$$

$$B_{1\tau} = \frac{4\pi}{C} j'\,. \tag{2'}$$

The physical essence of these equalities is simple: at the initial instant the induction lines $\mathbf{B}_1$ bend round the conducting body, i.e., the external magnetic field turns out to be tangent to the conductor (Fig. 1a), while around the body there occur brief changes in field $\mathbf{B}_1$, which induces an electric field E, under the action of which the surface current

$$\mathbf{j}' = \lambda' \cdot \mathbf{E}\,. \tag{3}$$

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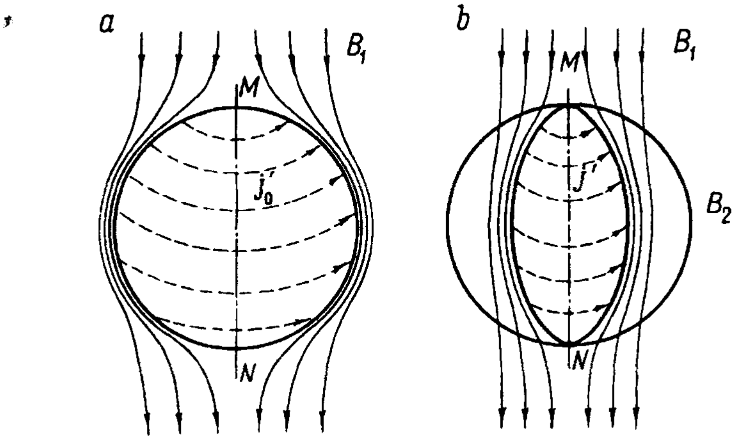

Fig. 1. Initial form of the lines of magnetic induction near body surface (a) and their path (b) after the spherical conductor is placed in a magnetic field.

flows along the conductor surface. If the body considered were a superconductor, the density of the surface current would remain invariant with time, and the magnetic field could never penetrate inside the body.

On the other hand, in the case of an ordinary conductor with electrical resistance, the surface current j' gradually becomes weaker because of dissipative losses, resulting, according to Eq. (2'), in a decrease in the tangential components of the magnetic force lines $B_{1\tau}$. Correspondingly, the earlier absent normal components of the lines of induction B_{1n} gradually increase. Simultaneously, according to Eq. (1), the normal components of the induction B_{2n} also increase, i.e., the magnetic field begins to penetrate inside the body. Geometrically, this process can be interpreted as conversion of magnetic force lines from those tangent to the conductor to secant ones, which penetrate deeper and deeper into it (Fig. 1b).

Comparison of Figs. 1a and 1b shows that at the initial instant of time the force lines bending around the conductor (having the shape of a sphere) passed along meridians on the spherical surface of the sphere; the dashed arrows show parallels along which the surface currents j' flowed. Figure 1b relates to a later instant $t > 0$, when magnetic force lines has already penetrated partially into the sphere and bend round the cavity, in which the field is still absent. This cavity has the shape of an ellipsoid of revolution elongated along the *MN* axis; along the parallels of this ellipsoid weakened currents j' flow. Later the currents j' become gradually weaker; the ellipsoid surface that separates the magnetized and unmagnetized portions of the sphere is contracted to the polar axis *MN*. This continues until the elongated ellipsoid degenerates into a segment of the *MN* axis and the magnetic field fills the entire volume of the conductor.

Assuming that the surface conductivity λ' is rather high and the density of the surface currents j' is appreciable, we have the right to conclude that the boundary ellipsoids release a large quantity of heat, and the temperature in these thin layers increases sharply. In this case, though metals have a high thermal conductivity, the thin high-temperature layer, while moving, does not spread, but moves as a solidified soliton. This phenomenon is a result of the high velocity of the magnetotemperature wave, which substantially exceeds the speed of temperature propagation due to thermal conductivity.

II. To justify the foregoing, we will derive a differential equation of the process of penetration of a magnetic field inside a conductor. For this purpose, we will make use of the Maxwell equations in a Gaussian system of units:

$$\text{rot } \mathbf{B} = \frac{4\pi}{C} \mathbf{j}, \tag{4}$$

$$\text{rot } \mathbf{E} = -\frac{1}{C} \frac{\partial \mathbf{B}}{\partial t}, \tag{5}$$

$$\text{div } \mathbf{B} = 0. \tag{6}$$

Substituting Eq. (3) into Eq. (4), we obtain

$$\text{rot } \mathbf{B} = \frac{1}{C} 4\pi \lambda' \cdot \mathbf{E}.$$

Operating with rot on both sides of this equality, we find

$$\text{rot rot } \mathbf{B} = \frac{1}{C} 4\pi\lambda' \cdot \text{rot } \mathbf{E}.$$

Taking into account the well-known identity rot rot A = grad·div A − ΔA and keeping in mind formulas (5) and (6), we obtain the equation

$$\Delta \mathbf{B} = \frac{1}{C^2} 4\pi\lambda' \frac{\partial \mathbf{B}}{\partial t} \tag{7}$$

with the initial condition

$$\mathbf{B}_{t=0} = 0. \tag{8}$$

As regards the boundary conditions, they are represented by equalities (1) and (2), but it must be kept in mind that the surface current density in Eq. (2) is a function of time $j' = j'(t)$.

We will write Eq. (7) in a somewhat converted form:

$$\frac{\partial \mathbf{B}}{\partial t} = \chi \Delta B, \tag{7'}$$

where

$$\chi = \frac{C^2}{4\pi\lambda'}. \tag{9}$$

We can easily see that vector equation (7′) is similar to the Fourier scalar heat-conduction equation

$$\frac{\partial T}{\partial t} = a \Delta T, \tag{10}$$

Here $a = k/cp$, where a and k are the coefficients of thermal diffusivity and thermal conductivity. It is therefore natural to call the quantity χ the coefficient of *magnetic conductivity.*

Owing to this analogy, it is possible, without solving Eq. (7′), to evaluate the velocity of penetration of a magnetic field inside a conductor. In fact, in heat conduction theory [5] it is shown that the time of equalization of the temperature in a body of size l is a quantity of the order of $\tau_T \sim l^2/a$, therefore, we may assume that the time τ_m of the penetration of a magnetic field into a spherical conductor of radius r is approximately equal to $\tau_m \sim r^2/\chi$. From this we may conclude that the velocity of the magnetotemperature wave is determined by the quantity χ, which is equal, according to Eq. (9), to the ratio of the constant $C_2/4\pi$ to the surface electrical conductivity λ'. In the aforementioned books on electromagnetism it is implicitly assumed that for all conducting media the coefficient λ' coincides with λ. One of the examples showing the inconsistency of this assumption is provided by superconductors. Their specific resistance is equal to zero; consequently, $\lambda = \lambda' = \infty$. At the same time, numerous experiments on measurement of the coefficient λ' indicate that the surface currents j' and electric conductivities λ' take finite values.

Our preliminary experiments on measurement of the coefficients λ' for various conductors show that the value of λ' for any conducting medium is a certain function of the surface layer thickness and temperature. At the present time we are investigating this problem in detail, but think it possible to use the approximate equality $\lambda' \approx \lambda$. Then, for example, for a copper sphere ($\lambda = 5.4 \cdot 10^{17}$ cgse units, $\chi = 150$ cm^2/sec, $a = 1$ cm^2/sec) of radius

$r = 12$ cm it turns out that the time of magnetic-field penetration τ_m is approximately equal to 1 sec, whereas the period of temperature equalization is 150 times greater.

It should be noted that for a number of substances (in particular, for a gas-discharge plasma) the values of λ', and, consequently, of the magnetoconductivity χ, increase sharply with temperature. As a result, the height of the soliton and the degree of retardation of its motion increase substantially, other conditions being equal.

Still more complex effects are observed if a conductor is placed in an alternating low-frequency magnetic field.

Conclusion. Analysis of the electrodynamic conditions at the boundary of contact of a conducting medium with a magnetostatic field leads to the conclusion that a superheated boundary layer appears on the medium surface. This hot thin layer moves inside the conductor with a velocity inversely proportional to the electrical conductivity of the medium λ and forms a decaying two-dimensional temperature soliton.

The aforementioned effect is especially significant in media whose electrical conductivity increases sharply with temperature (for example, gas-discharge plasma with $\lambda \sim T^{3/2}$). In this case the soliton thickness contracts, while its height increases. Simultaneously, the velocities of the soliton and of the accompanying magnetic field penetrating inside the conductor slow down.

NOTATION

B_1 and B_2, external and internal magnetic fields; B_n and B_τ, normal and tangential components of magnetic fields; j', density of the surface current; C, electrodynamic constant (velocity of light in vacuum); λ, coefficient of electrical conductivity; λ', surface electrical conductivity; Δ, Laplace operator (Laplacian); c and ρ, heat capacity and density of substance; k and a, coefficients of thermal conductivity and thermal diffusivity; T, temperature.

REFERENCES

1. I. E. Tamm, Fundamentals of the Theory of Electricity [in Russian], Moscow-Leningrad (1946).
2. L. D. Landau and E. M. Lifshits, Electrodynamics of Continua [in Russia], Moscow (1957).
3. A. N. Matveev, Electrodynamics and the Theory of Relativity [in Russian], Moscow (1964).
4. I. E. Irodov, Basic Laws of Electromagnetism [in Russian], Moscow (1983).
5. H. Gröber, S. Erk, and U. Grigull, Die Grundgesetze der Wärmeübertragung, Springer-Verlag, Berlin (1955).